Exercise 66

For the following exercises, find the point of intersection of each pair of lines if it exists. If it does not exist, indicate that there is no point of intersection.

$$2x - 3y = 12$$
$$5y + x = 30$$

Solution

Solve the two equations for y.

$$\begin{cases} -3y = -2x + 12\\ 5y = -x + 30 \end{cases}$$
$$\begin{cases} y = \frac{2}{3}x - 4\\ y = -\frac{1}{5}x + 6 \end{cases}$$

Because the two lines have different slopes, there is a point of intersection. Set the two equations equal to each other and solve the equation for x.

$$\frac{2}{3}x - 4 = -\frac{1}{5}x + 6$$
$$\frac{2}{3}x + \frac{1}{5}x = 4 + 6$$
$$\frac{13}{15}x = 10$$
$$x = \frac{150}{13}$$

Now plug this value of x into either of the two equations and get the corresponding y-value.

$$y = \frac{2}{3} \left(\frac{150}{13}\right) - 4 = \frac{48}{13}$$

Therefore, the point of intersection is

$$\left(\frac{150}{13},\frac{48}{13}\right).$$

